Network Structure and the Division of Labor

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The division of labor in society is a classic problem of social coordination. In *the Wealth of Nations* (1776), Adam Smith introduced the example of a pin factory to motivate the beneficial effects of trade between nations. These benefits lay in a division of labor between nations where nations specialize in the production of goods for which they hold an absolute advantage. Much of the book was used to convince government actors to allow the free flow of trade so that the benefits of an international division of labor could be realized. Without the right national trade policies, Smith recognized that the division of labor – and the prosperity that might follow from it -- could be stifled. Where Smith saw the emergence of a division of labor between nations largely as a regulatory problem involving taxes and tariffs, a century later Emile Durkheim considered the possibility of non-governmental social barriers to the emergence of specialization. In his work *The Division of Labor in Society* (1893), Durkheim poses a classic puzzle of sociological theory, how does specialization, the cooperative interdependence of organic solidarity, emerge from generalist, communities he described as being characterized by mechanical solidarity.

Durkheim's answer, while difficult to reduce to a series of hypothesis (Gibbs 2003), contained a formalist structural element. Increasing density of interaction was one of the preconditions he listed for the transition to the cooperative interdependence of the division of labor (Durkheim 1996: xx). This assertion is interesting as it stands in some contrast to another classical theorist's description of the transition to modernity. Ferdinand Tönnies described the specialization, industrialization, and marketization of society as *gesellschaft*, a society based on non-personal, contractual relations (2011). In structural terms, this corresponds roughly to a shift from Colemanesque intensive social capital, that relies upon cohesive groups, to the more flexible arm's length relations of Mark Granovetter's weak ties (1973) or Ronald Burt's structural holes (1995). Thus, classical theory gives rise to different hypotheses regarding the optimal structure of relations for encouraging the development of specialization.

The problem is not, however, confined to history or theory. The problem of collectively coordinating specialization reoccurs as firms jockey for monopoly control over specialized niches within larger industries (White 1981) as well when they attempt to develop distinct brand identities. Changes in connectivity between communities produced through the creation or destruction of highways, seaways, and other transportation routes can directly impact specialization rates, as has been seen for example in the effects of the construction of the Inter-Oceanic Highway in the Amazonian basin (Perz et al. 2013). And patterns of friendship have been observed to impact the specialized production of medicinal herbs in home gardens in remote areas (Diaz-Riviriego et al. 2016). The field of economic sociology has provided ample evidence that exchange is constrained by pre-existing patterns of interaction, whether these are determined by proximity, transportation infrastructure, river networks, or affectual bonds of trust, loyalty, or homophily. We consider here how these durable patterns can impact the development of complementarity that lies at the heart of profitable exchange and market expansion.

In order to investigate these issues, we conduct agent-based simulations (Macy & Willer 2002) from within the *graph-coloring* paradigm (Jensen & Toft 2011) to model and analyze the conditions under which specialization may or may not emerge within a networked population of generalist producers and consider the different conditions that inhibit or encourage this transition.

While such models are abstract, there is a robust tradition in sociology of employing formal models to produce valuable insights into social phenomena (Granovetter 1978, Feld 1981, Watts 1999, Centola & Macy 2007, for example). Using computational models that vary and compare different network structures of agents attempting to coordinate, we find that network structure does indeed have an effect on the likelihood and rate of specialization. Network structure can produce *social guides* that facilitate the emergence of the division of labor, but can also produce *social traps* that lead to suboptimal specialization rates. In our models, specialization is strongly impacted by the capacity to store surplus goods, which mediates the impact of network structure by transforming social guides into social traps. In both cases, the number of graph-coloring solutions in a network, referred to as the chromatic polynomial, has a stronger impact on specialization rates than other conventional network statistics.

The findings suggest a revision of our theoretical understanding of the conditions that foster the development of a successful division of labor. They support previous work in showing that network structure plays an important role in determining the likelihood and pace of the development of specialization; however, the structural features previous theorists focused on -- cohesion and arm's length ties – may be ancillary to the transition. Instead, the solution set, i.e. the chromatic polynomial, appears to serve as a previously hidden sub-stratum constraint on the emergence of specialization in the formal models. Because the chromatic polynomial is sensitive to small changes in network structure, this further suggests that dramatic changes in the likelihood of coordination may be brought about by minute structural adjustments. It should also be noted that these findings are very different from the results that have been produced by formal models of diffusion, in which weak ties play a large role. This difference is appropriate given that the underlying models are entirely distinct processes. It also highlights the potential non-transferability of insights from diffusion research to the problems of coordination. Since, the processes of diffusion have arguably received the lion's share of attention in social network research, coordination problems may profit from more attention.

Social structure and the division of labor:

Adam Smith has long been recognized as the pre-eminent theorist of the division of labor. Smith, however, was not concerned with the problem of how specialization develops over time as with promoting its benefits to society and the state. Smith assumed that knowledge of the benefits of the division of labor would be enough to encourage specialization. For him, and for many of those who followed him, the limits of specialization were set by the size of the market (Becker and Murphy 1992).

By the nineteenth century, social observers were finding signs that market expansion and increasing specialization were accompanied by other fundamental changes in social structure. As noted above, Durkheim suggested that an increasing density of interactions played an important role in his theory of the change from homogenous, independent producers to interdependent, specialists. Ferdinand Tönnies famously characterized a transformation from *gemeinschaft* to *gesellschaft* (2011), in which small homogenous communities bound by personalistic ties based in natural will became larger, more complex societies bound by contractual and impersonal ties based in rational will. Similar arguments about the loss of community ties in market society are made by Karl Polanyi in *The Great Transformation* (2001). The influence of these two thinkers alone was enough to turn the decreasing density of community ties into a major narrative of the transition into modernity, industrialization, and capitalism. However, ambiguity remains in these accounts as to what is a cause or consequence and the importance of other major historical changes accompanying the reorganization of social relations.

In the twentieth century, sociologists continued this tradition of inquiry through conceptual development (Kempner 1972, Blau and Scott 1962), measurement (Gibbs and Browning 1966, Clemente and Sturgis 1972, Gibbs and Poston 1975), and even experimental studies (Thomas 1957, Weick 1965). Here we return to the theorization of the social conditions that encourage specialization by focusing on network structure. While work in economics has incorporated the idea of coordination costs as an additional complicating factor affecting the development and expansion of specialization (Becker and Murphy 1992) and heterodox economists have considered the role of social interaction in production decisions (Foley 2019), recent research has done little to mobilize our improved understanding of network structure and dynamics to consider how variation in network topology can affect the process of specialization.

Our expectations are in line with previous work in so far as we expect structure to matter for the outcome of interest, but our expectation is that the impact of structure may be further refined by drawing further from recent research social network theory and formal approaches to decentralized coordination. In particular, recent research has been able to combine network and game-theoretic approaches to decentralized coordination problem through the model of the graph-coloring problem (Jensen & Toft 2011). Kearns, Suri, & Montfort (2006) showed that experimental subjects were better able to solve cycle graphs than preferential attachment networks. McCubbins, Paturi, and Weller (2009) showed that increased density of connections in several stylized graphs helped experimental subjects find successful solutions. And Shirado and Christakis (2017) showed that adding noise via pre-programmed bots can help experimental subjects in their efforts to solve the coordination problem.

One element that remains in the background of these important studies, is that the likelihood of solving the graph-coloring problem is strongly related to a measure less common to social network research: the chromatic polynomial. The chromatic polynomial is the number of solutions that exist to the graph-coloring problem. We expect that the number of solutions that exist for decentralized coordination of complementarities in exchange is going to have a large effect on the likelihood of the emergence and spread of a division of labor.

Another way to put this is that the benefit of the division of labor comes from exchanging different goods. As has been noted, there is no reason for either a dog or a person to exchange a bone for a bone (Bearman 1997:1388). For the division of labor to be profitable, producers must exchange goods with people the produce different goods from themselves. If everyone specializes in bones, there are no gains to trade. If it is easy for people to find partners that produce different goods, it is more likely that they will specialize and enter into a productive division of labor. If it is difficult for them to find people that produce different goods, it seems much less likely that they will specialize and the division of labor is correspondingly less likely to emerge. The chromatic polynomial is the number of solutions to this patterning problem, thus there is every reason to believe it would have a large impact on specialization rates.

Our analysis suggests that structural constraint may have two types of effects. We understand structural constraint to exist when networks are organized in a rigid pattern with little to no randomization or when the number of possible solutions to the graph-coloring is low. Structural constraint is therefore high in the simple, repeating pattern of a lattice network and inversely manifested in the chromatic polynomial. Structural constraint can produce a negative impact by creating *social traps*; it can also have a positive impact by producing *social guides*.

Social traps are the more intuitive concept, so we will begin there. In networks with lower chromatic polynomials, there are less solutions to the complementarity problem. It follows that a smaller number of decisions by actors will lead to complete specialization and more of the set of possible decisions will lead to less than complete specialization. In the case of food, water, and shelter, if you want to specialize in food, only a small number of arrangements will allow you to also procure water and shelter from your neighbors. The network is like a maze filled with dead ends, and the large number of incorrect solutions (or suboptimal solutions) act as a social trap (Shirado and Christakis 2017).

Despite this, there are conditions under which a constrained pathway may serve as a guide to finding a solution. It is an easy decision to specialize in food if you are connected to only two neighbors, one of which specializes in water and the other in shelter. In a simple lattice network that has a low chromatic polynomial, that pattern of ties provides a constraint that makes it easy for actors to choose the correct good to solve the complementarity problem. Conversely, no specializations emerge among isolated individuals. Somewhat counterintuitively, if many solutions are viable, each local condition may be too open to present a clear solution. In these cases, a lower number of solutions is likely to lead to a higher rate of specialization as decentralized decision-makers are forced into a narrow pattern of repeated complementary exchange. Interestingly, tightly constrained patterns of exchange are found in many traditional exchange systems, such as for example the Kula Ring of the Trobriand Islands (Malinowski 1984, Strathern 1971, Schieffelin 1981).

Which condition obtains will depend upon the possibility of specializing a certain commodity before other exchange partners have chosen their specialization. We call this capacity partial specialization. If partial specialization is possible, actors that have found different local solutions to the complementarity problem have the ability to readjust if there solutions are not globally optimal. They can continue to experiment, but a higher number of solutions speeds the successful resolution of their experimentation. In these cases, networks with many global solutions facilitate development of optimal specialization, and the constraining effect of structure has a negative impact and functions as a *social trap*. Conversely, if partial specialization is not possible, actors have less ability to experiment. If actors do not have a clear choice, they will not choose to specialize as they cannot be sure that they will be able to exchange with others for the goods that they need in order for specialization to be profitable. In these cases, low constraint leads to impasses in the specialization process and high structural constraints can serve as *social guide* to successful global specialization.

Partial specialization may be thought of as the capacity to store goods. If actors can store goods, they can put aside a specialized commodity for exchange with a partner who may choose a different specialization in future rounds. In that case, the solution can be found stochastically over rearrangements of specialization patterns. But, without the capacity to store goods, actors have to exchange goods simultaneously just after production. In this context, actors have to wait to choose a specialization until their partners have already specialized in order to be sure specialization is of value. Each actor has to find a local solution to the complementarity problem in order to specialize, and the complementarity problem has to be solved in a deterministic way.

We also find that the capacity to store goods is conducive to the emergence of the division of labor when networks increase in complexity, meaning when they have lower structural constraint and patterns of relations do not serve as a legible social guide. Given that property and property rights increase individuals' ability to safely store goods, this effect appears to offer another interpretation for the positive association between the institutionalization of property rights and economic development (North 1990 1994, Rodrik et al. 2004, Acemoglu, Johnson, and Robinson 2001, 2002, Knack and Keefer 1995, Hall and Jones 1999). Property may make it easier for groups to solve the collective problem of specialization.

Formalization of the division of labor problem:

To explore the dynamics of specialization, we use the graph-coloring game. The graph-coloring game was originally motivated with reference to a small number of real-world problems and familiar dilemmas including the perennial problem of scheduling class times, choosing differentiated ring tones, selecting a frequency in broadcasting systems, or choosing an area of expertise to cultivate within an organization (Kearns, Suri, & Montfort 2006, Shirado and Christakis 2017). The game was meant to represent in a general and abstract way the larger class of social situations in which individuals engage collectively in distributed problem-solving with restricted, local information. In this sense, the game addresses in a slightly different way the same theoretical problems debated in the 'collective intelligence' literature and one that underlies many assumptions about market properties: Can decentralized individuals collectively resolve difficult social coordination problems?

The graph-coloring game begins with a network of interconnected nodes. Nodes in the network may take on one of a defined set of colors. The goal is for each node to take on a different color than its neighbors (Kearns, Suri, & Montfort 2006). In this way, the game effectively captures the fundamental features of the problem the division of labor: interdependence and specialization.

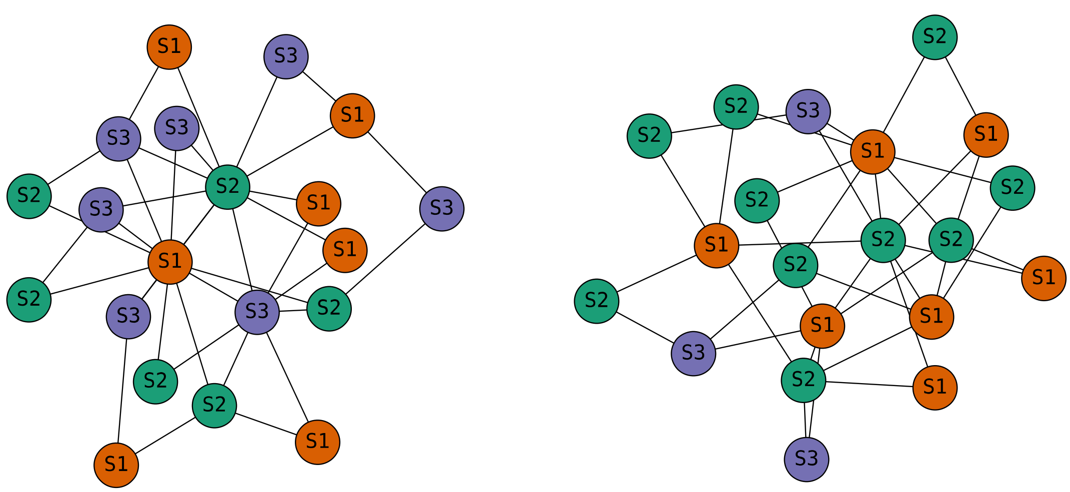
This characterization follows if one begins with the proposition that human beings have more than one basic need. At a minimum we might begin by assuming that humans require food, water, and shelter. Before specialization, individuals are necessarily generalists. Each individual must work to procure the food, water, and shelter necessary for their own existence. They cannot survive without all three items. In order to specialize in any one good, they must be able to acquire the other goods from someone else. If one person gathers food, someone else must procure adequate water supplies, and another must construct, or maintain a shelter from the elements. The individuals must coordinate their activity with each other to survive and prosper, and the decision process is distributed across actors.

Two measures are important to understanding the features of the graph-coloring game. The chromatic number of a graph is the minimum number of colors for which a solution to the game exists. If there are too few numbers, a solution will not be possible. We only consider instances in which a solution is possible. The structure of the network can alter the minimum number of colors necessary to solve the game. For example, in a connected ring cycle in which each node has two neighbors and there is an even number of nodes, the chromatic number is two. A solution is possible if nodes alternate between colors. A more complex random network of the same size may require more colors for a solution. In general, the chromatic number will vary depending on the number of ties between nodes and the topological properties of the network.

The second measure is the chromatic polynomial, which is the number of optimal color combinations for a given network with the chromatic number in which no node is connected to a node of the same color (i.e., optimal solutions for graph coloring). It follows that a high chromatic polynomial indicates a large number of optimal solutions and a low chromatic polynomial indicates few solutions.

The number of solutions to the game varies depending upon the features of the network, but those features can be difficult to detect using descriptive structural measures other than the polynomial itself. For illustration, Figure 1 presents two networks generated through simulated processes of preferential attachment (Barabási and Albert 1999), a standard generative model in social network research. The networks were generated through the same random preferential attachment process with 20 nodes. They have the same number of nodes, the same chromatic number, and the same density. However, the chromatic polynomial of the left network is 12 and that of the right is 8,280. Small and difficult to observe changes in network topology produce huge variance the size of the set of possible solutions.

Figure 1. Two preferential attachment networks with two levels of chromatic polynomial



Chromatic polynomial = 12 | Chromatic polynomial = 8280

In the original application of the graph-coloring game, individuals were incentivized to identify solutions to the game externally. Experimental participants were given a small sum of money for each time they successfully identified a solution for a specific network (Kearns, Suri, & Montfort 2006, Shirado and Christakis 2017). In order to apply the game to the problem of the division of labor, which is an economic problem, the incentives needed to be reconfigured so that they more closely modeled exchange processes. We adjusted the game by incorporating a pay-off structure in which individuals have a choice to generalize or specialize, but they receive a greater benefit from specialization given that their other needs are satisfied by exchange with others producing the remaining goods that they require (Foley 2019).

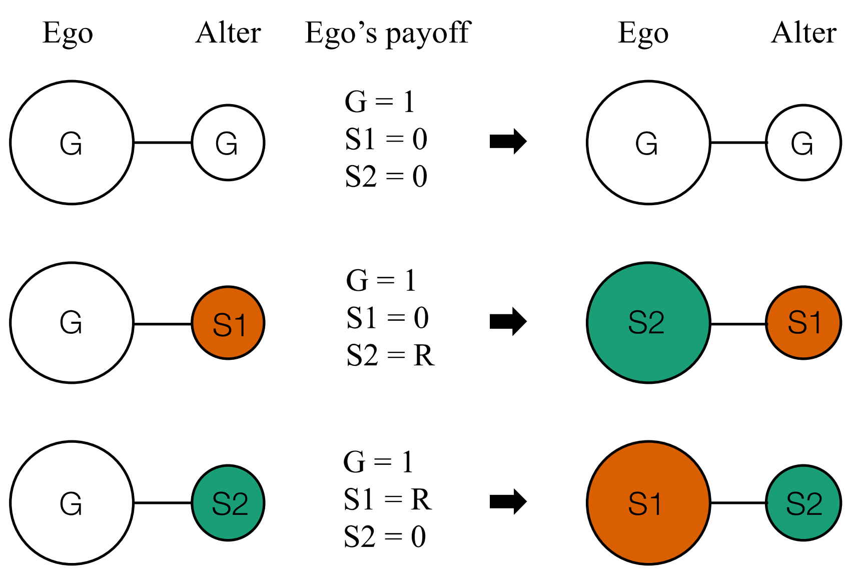
We begin with the assumption that all nodes are producers. In order to imitate conditions that encourage the specialization of labor, we do not impose caps on productive capacity, so nodes may exchange with any number of partners. There are no brokers, which follows because brokerage is in itself a special kind of specialization. The pay-off structure for two-player game with two items is represented in Figure 2A. Choosing generalization (G) benefits 1 regardless of alters' status. Specialization of Item 1 (S1) and Item 2 (S2) gives ego more benefit (R > 1) only when their alter specializes the other item; otherwise ego earns nothing (Figure 2B).

Figure 2. Pay-off structure and specialization development with one neighbor and two items

A

		Alter		
		G	S1	S2
	G	1	1	1
Ego	S1	0	0	R
	S2	0	R	0

B



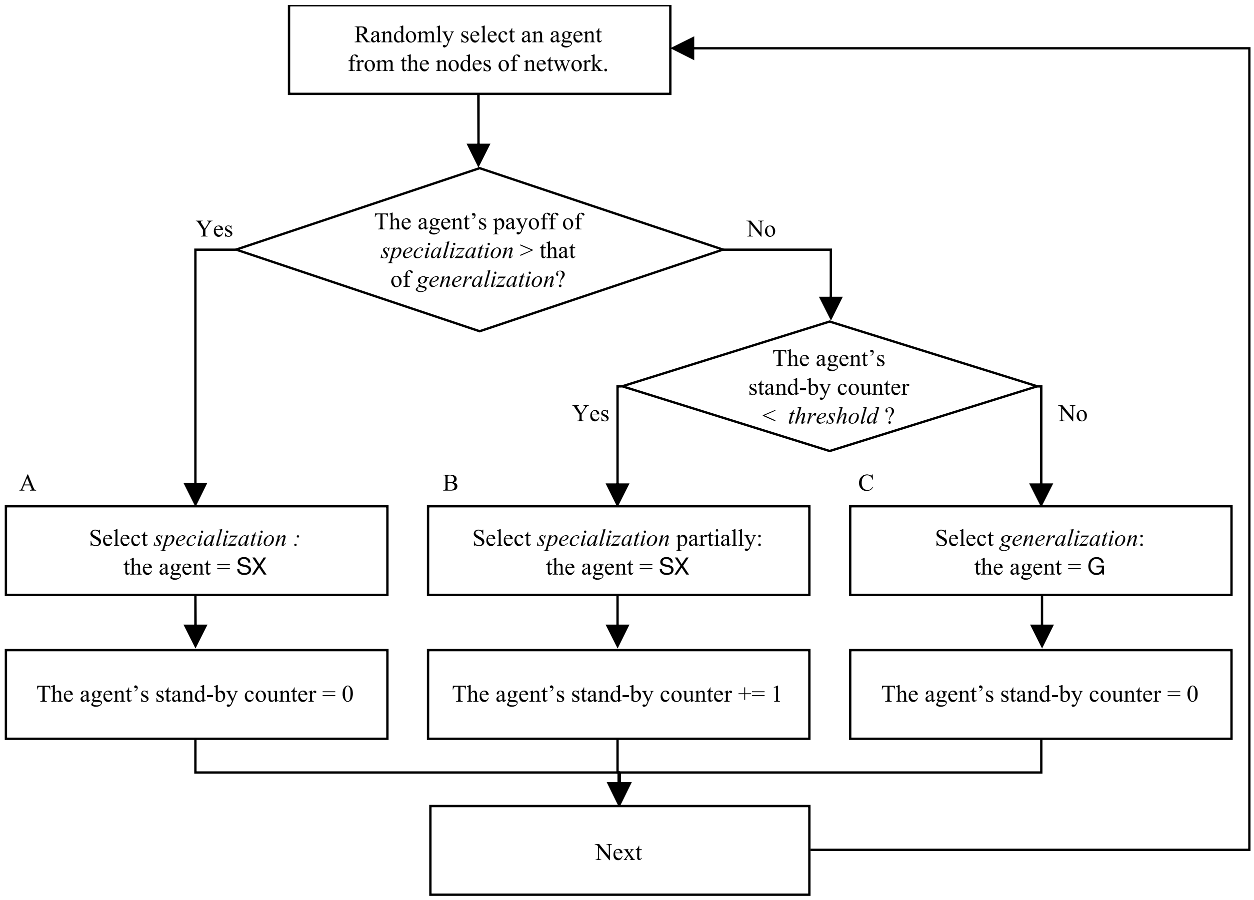
The outcomes are indicated by the color of the nodes. Colors correspond to the type of specialization (orange is for S1 and green is for S2). White signifies generalization – i.e. lack of specialization.

Structural constraint as social guide and social trap

The impact of structure may be explored by putting nodes into larger networks of interdependency and exchange. Understanding the decision process for nodes at the micro-level of small networks helps to explain the malleable role of constraint under different conditions. We consider two types of models based on the pay-off structure described in the previous section. In the first, specialization pays off only if the node is connected to another node of a different color. In the second, we relax the condition that requires simultaneous specialization by allowing nodes to store specialized goods for exchange with other nodes in later rounds of play. We refer to the more stringent condition as a "complete-specialization" process and the less-stringent condition as a "partial-specialization" process.

Figure 3 presents the way in which partial specialization is incorporated into the decision process as a threshold of goods that may be stored. We control the level of stringency with the threshold for how long actors can wait to earn specialization benefits. If the threshold is 0, each actor selects specialization only when the payoff for specialization is better than that of generalization under the current local condition (i.e., Process A or C in Figure 3; complete specialization only). If the threshold is significantly larger, each actor selects specialization of an item even when the current local condition does not give the actor specialization benefits (i.e., Process A or B in Figure 3; partial specialization allowed).

Figure 3. Flow chart of graph-coloring game in agent-based simulations with partial specialization



The threshed in our simulations can be regarded as storage capacity for actors. There are many possible empirical correlates. Storage capacity may be thought of as something good specific, fish for example is more difficult to preserve than grain, technological, the invention of refrigeration vastly expanded societies capacity to store food products, or as surplus wealth that allows for innovation, such as funding for research labs in for-profit firms. In all cases, however, storage implies the recognition of the idea of property protections. Individuals must have the capacity to reserve some goods for exchange with others. This capacity does not exist – or has a tenuous existence is societies without formal or normative protections for private property.

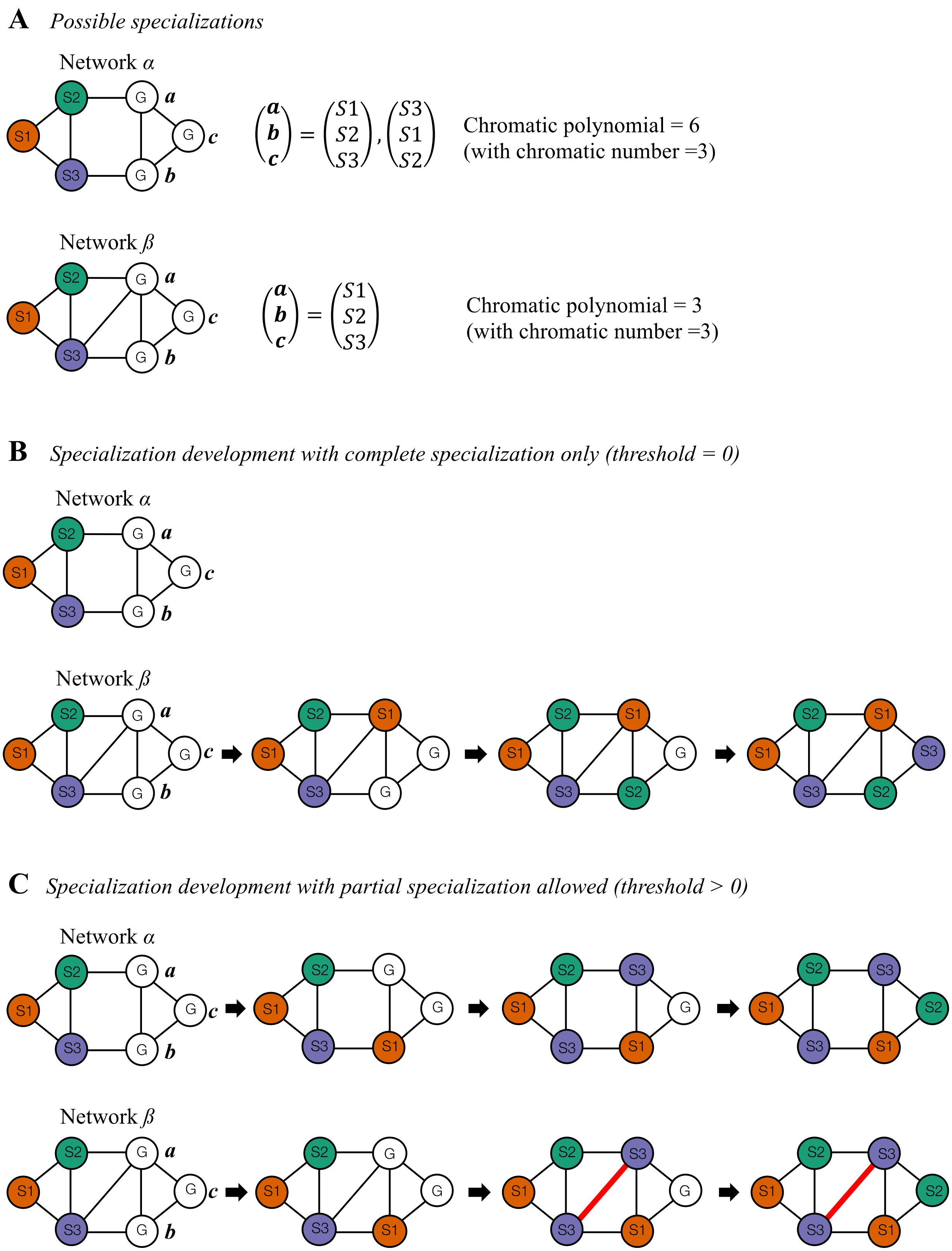
Figure 4 illustrates how property rights can complicate the effect of structure on the likelihood of spread of specialization. Figure 4A illustrates two different networks of six nodes. Both Network alpha and beta have a chromatic number of 3. This number indicates that there must be at least three specializations from which nodes may choose in order for total specialization to occur. Network alpha has a chromatic polynomial of 6, meaning that there are six possible arrangements in which total specialization can occur. Network beta has one additional tie (i.e., additional structural constraint) that halves the chromatic polynomial of the network.

In each of these simplified simulations, we begin the process with a small triad of specialized nodes. In the example of Figure 4A, when the triad has specialized, the other three agents (Agents *a*, *b*, and *c*) have two possible ways to successfully specialize in Network alpha while they have only one combination in Network beta because of the additional structural constraint. This difference is discrepancy reflected in the chromatic polynomial.

Figure 4B represents the possible stages of play under the conditions of no property (i.e., the threshold = 0 in Figure 3). Network alpha, with a chromatic polynomial of 6, has no iterations of play because the agents are unable to find a solution. Agents *a* and *b* are each connected to a specialist, but they are only connected to one specialist. Thus, if Agent *a* or *b* choose to specialize, they will not be able to procure one of the goods they require to benefit at that time. There is therefore no incentive to specialize, and the division of labor fails to spread through the network.

The additional tie in Network beta creates a structural difference that produces a very different outcome. In this network, Agent *a* is connected to two specialists producing different goods. The benefit to specialization is clear, and Agent *a* chooses its color (for this example, S1). The specialization of Agent *a* creates a situation in which Agent *b* is now tied to two specialists of different goods. Once again there is an immediate benefit to specialization for the agent. The same process then repeats for Agent *c*. In fact, if one imaged this network of six as one chain in a circular lattice network, it follows that specialization would spread throughout the network as agents followed the easily discernable pattern. In this case, the constrained choice set given to agents encourages the spread of the division of labor. The network with the lower chromatic polynomial, Network beta, has higher rates of specialization – even though there are fewer global optimal solutions. In the no-property setting, the constraints imposed by the structure of the network serves as a social guide to agents that encourages specialization.

Figure 4. Possible outcomes for simple networks of six nodes



This process can be compared to the decision sequence that unfolds in Figure 4C, where agents are allowed to hold onto property past one round of play (i.e., the threshold in Figure 3 is large enough). In this set up Agents *a* and *b* in Network alpha are able to overcome the difficulty of simultaneously locating both required goods. For example, Agent *b* can choose a specialization although it has only one specialist at the time. Agent *a* is able to then choose a complementary specialization and the division of labor spreads through the network in a few short steps. In contrast, in Network beta, the solution set is more constrained, making it more difficult for nodes to choose the right specialization. When Agent *b* chooses the specialization of Item 1 earlier than Agent *a*, it makes a local conflict (indicated with red line in Figure 4C) and blocks a path to the globally optimal solution. Once the agents are all specialized, any single change cannot resolve the suboptimality. In the context with property, the constrained network structure instead serves as a social trap that allows agents to choose local solutions that are suboptimal from the global standpoint. Thus, the structural constraint of the chromatic polynomial can serve as either a social guide or a social trap depending on the existence of property rights.

Simulation Process:

To explore the effect of network topology on the successful coordination and the achievement of a mutually beneficial division of labor between producers, we conduct agent-based simulations incorporating the graph-coloring game (Figure 3) in a fixed network structure. Agents, that are assigned to each node of a network, calculate the payoff with the specializations of their network neighbors to update their own node color (i.e., specialization of an item or generalization; Figure 2). They are not able to reconfigure their ties because doing so would change the structure of the network, making it impossible to measure network structure as a constant affecting the outcomes of the coordination process.

In the simulation, we first generate a network. Within this network, a small cluster of nodes are randomly assigned to an initial state of specialization.[1] That is, we randomly select a dyad in a network and assign different colors to the two nodes for two-color games if the network's chromatic number is 2. We color a triangle for three-color games, and a four-node complete subgraph for four-color games. With this initial setting, we examine the evolution of specialization in a network, i.e., how far a minority of pioneers employing specialization can invade the population of generalists (Nowak and May 1992, Szabó and Fáth 2007).

Figure 3 presents the simulation process beginning at this point. One agent is chosen at random to update its state based upon the payoff it could receive from its neighbors. When private property is not allowed (i.e., the threshold = 0) or the agent's stand-by counter reaches the threshold, the agent chooses specialization or generalization simply with the expected payoff. For example, in a three-color game, the agent chooses the different color from its neighbors when its neighbors have chosen two different colors (i.e., the specialization of the complementary item; Figure 3A); otherwise, it chooses white (i.e., generalization; Figure 3C). When private property is allowed under partial specialization (i.e., the threshold > 0) and the agent's stand-by counter is less than

[1] For robustness, we also ran models in which generalization and specialization were randomly assigned across nodes at the initial state, and a model in which specialization was irreversible.

the threshold, the agent follows a simple greedy strategy; it chooses a color that minimizes the overlap with the colors of its neighbors (except for white; Figure 3B) (Chaudhuri, Chung & Jamall 2008). After the color update, when the agent's specialization is incomplete (i.e., the agent does not have enough specialized neighbors at the time), its stand-by counter is incremented by one; otherwise the counter is reset to zero.

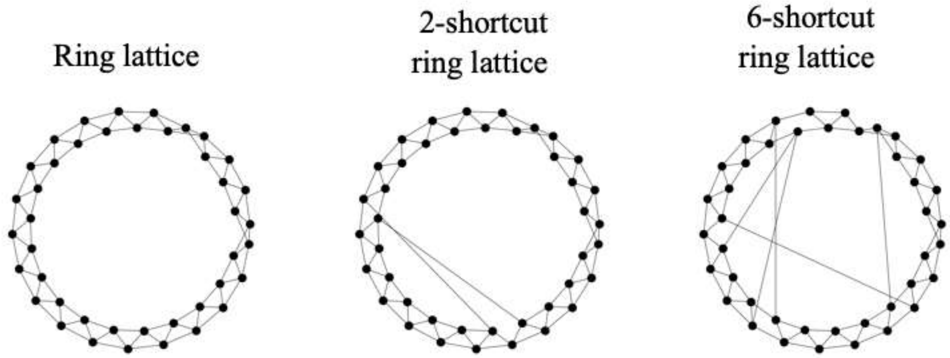
This step is repeated 100,000 times unless the network reaches a configuration in which no agents can change their state. At the end of the iterations through the decision process, we calculate the proportion of specialization that occurs in the network. We then repeat the process 100 times using different random sequences of agents' decision moments including the initial color assignment. One observation is the average proportion of 100 realizations of the same network with different update sequences. We tested 500 networks for each network model. The exception to this process was the ring lattice network which has no variation in outcomes because of its particular network topology.

Results for Small-World Networks:

First, we consider small world networks and the impact of random ties. Small world networks are highly clustered networks with low average path length. The combination of clustering and reachability is created by the addition of a low proportion of random connections. These random connections typically reach across the dense, local clusters to link connected otherwise distant nodes (i.e., "shortcuts"). In doing so, they dramatically decrease the time it can take for information or various forms of contagion to spread through the network (Watts 2006). The random connections that span across small world networks are conceptually related to weak ties (Granovetter 1973). Small worlds can in fact be characterized as worlds bound together by weak ties; indeed, one way of reading the theory of small worlds is as a mathematical demonstration of the importance of weak ties at the macro-structural level.

Weak ties are particularly interesting in terms of evaluating the emergence of the division of labor because of their association with the onset of modernity. As noted previously, closely-bounded community and kin ties have been associated with pre-industrial societies where arm's length ties have been associated with modern market expansion, also suggesting an increase in specialization and the spread of the division of labor (Tönnies 2011). Based on these loose historical generalizations, one might expect weak ties to encourage the division of labor. Our findings, however, do not support this expectation.

Table 1. Descriptive statistics for small world networks



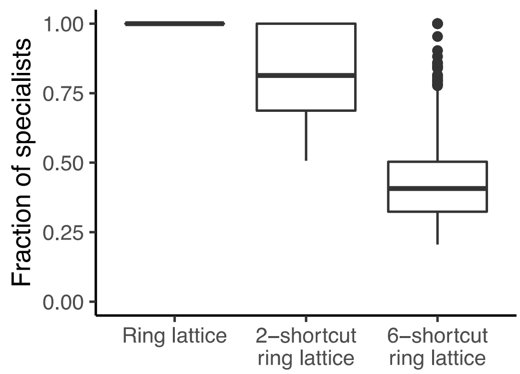
	Ring lattice	2-shortcut ring lattic	6-shortcut ring lattice
Number of shortcuts ………….	0	2	6
Rewiring probability ………….	0.000	0.024	0.071
Number of nodes ……………..	42	42	42
Number of edges ……………..	84	84	84
Density ………………………	0.098	0.098	0.098
Chromatic number ……………	3	3	3
Chromatic polynomial ……….	6	6	6
Clustering coefficient ………..	0.500	0.469	0.425
	(n/a)	(0.012)	(0.021)
Average shortest path length …	5.634	4.994	4.229
	(n/a)	(0.353)	(0.350)

Table 1 presents descriptive statistics for the small-world network simulations. All networks have 42 nodes and 84 edges. The density of networks does not vary because shortcuts are created by replacing existing links (Watts and Strogatz 1998). Each node has four links for all networks. The chromatic number is 3 for all networks, and the chromatic polynomial, i.e. the number of solutions, is 6 for all networks. The random rewiring of ties used to generate small-world networks provides some variation in the clustering coefficient and shortest path length across the different realizations of the network type.[2] These types are simulated 500 times. There is no variation in the lattice structure, so the standard deviation is not reported. The clustering coefficient and average shortest path length both decrease with higher rates of rewiring.

[2] It is noteworthy that, to clarify the small-world effect, we control the network variation so that all the networks' chromatic polynomial is 6 with 3 colors. Without control, the chromatic polynomial of small-world networks would vary (e.g., no solutions with three colors). We also create the networks with the exact number of shortcuts (two or six) which is different from a typical graph generator of small-world networks using rewiring probabilities (Watts and Strogatz 1998).

Figure 5. Small-world network simulation results

A *Complete specialization only (threshold = 0)*



B *Partial specialization allowed (threshold = ∞)*

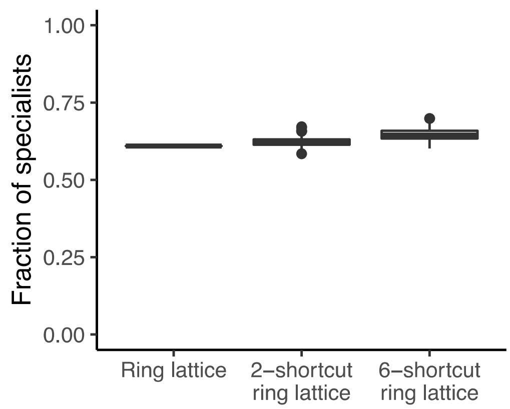


Figure 5 presents the results of the simulation for the small-world networks. Panel A presents the results for the condition in which nodes have no storage capacity and panel B presents the results for the condition in which nodes do have storage capacity. In Figure 5A, it is clear that random ties and their related structural effects, i.e. lower clustering coefficients and lower average path lengths, are associated with differences in the rate of specialization. However, the direction of the association does not follow that suggested by prior theory. The networks with the highest rate of rewiring also have the lowest rate of specialization, and even a small amount of rewiring shifts the average rate of specialization down by a little less than 25%.

This result can be understood by considering the way in which simple network structures can serve as a social guide to actors. The ring lattice is a simple model with a chromatic number of 3 and a chromatic polynomial of 6. For a network of 42 nodes, this is low number of solutions. However, the patterned nature of the connection in the network make finding a solution at the local level trivial for agents even when they cannot see past their local environment. As a result, there is a 100% solution rate. In contrast, adding random connections decreases the specialization rate. Adding two shortcuts that span the network decrease the average path length, but also decrease the fraction of the population that are able to successfully adopt specialization. This decrease is the result of the additional complexity introduced by random connections that disrupt the otherwise clear lattice-like pattern of on and off again product choice. Adding additional shortcuts, i.e. increasing the likelihood of rewiring, further decreases the proportion of the population that chooses to specialize.

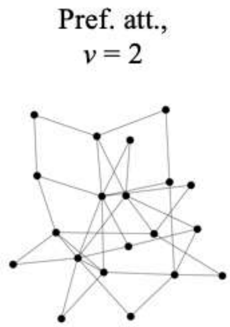
When partial specialization allowed agents to preserve property between rounds, the negative impact of shortcuts disappears (Figure 5B). Agents can choose specialization without the pre-specialization of their neighbors, and therefore do not require the structural guide for specialization. The result shows that adding shortcuts has a very minimal, if positive, impact on the development of specialization. In other words, while shortcuts impair the structural effect of social guide in the no-property world, they do little to alleviate the negative impact of social traps in the property world. Furthermore, in these highly organized lattice-based networks with two or less shortcuts, the capacity to store property is associated with a decrease in rates of specialization. In these cases, the property rights can actually decrease the development and spread of the division of labor. As reported in the descriptive statistics, these changes in specialization rates are not related to differences in the density of the network, the size of the network, or the chromatic polynomial of the network.

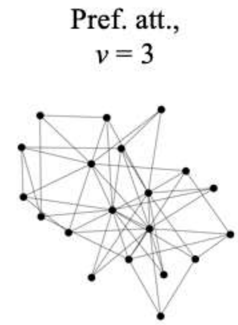
Results for Preferential-Attachment Networks:

The effect of the chromatic polynomial on the possibility of specialization is difficult to gauge in the small-world networks because its variation is constrained by the low levels of randomization introduced by the rewiring of lattice networks. In order to explore the effect of the chromatic polynomial on specialization rates, we turn to another commonly observed and theorized network type: preferential attachment models.

Preferential attachment networks capture a common social phenomenon when the likelihood of having a connection is conditioned on the number of existing connections. They capture, for example, instances in which having many friends makes it more likely that an individual will make new friends or when having a higher number of twitter followers increases the likelihood that an individual will attract new twitter followers. This effect produces networks with uneven degree distributions. We use the Barabasi-Albert model (Barabási and Albert 1999) where v is the parameter that maps the nodal degree to the likelihood of adding a new connection. The Barabasi-Albert model differs from small world models in that the network is generated by adding new nodes and ties rather than rewiring existing ties between a fixed set of nodes. One of the results of this generation process is greater variation in the chromatic polynomial.

Table 2. Descriptive statistics for preferential-attachment networks





	Pref. att., v = 2	Pref. att., v = 3
Number of edges to attach …….	2	3
Number of nodes ……………..	20	20
Number of edges ……………..	36	51
Density ……………………….	0.189	0.268
Chromatic number ……………	3	4
Chromatic polynomial ……….	421.5	43805.0
	(507.3)	(58075.8)
Clustering coefficient ………..	0.322	0.393
	(0.110)	(0.075)
Average shortest path length ….	2.156	1.866
	(0.072)	(0.036)

Table 2 presents the descriptive statistics for the preferential-attachment networks. In Table 2, *v* represents the number of edges to attach in preferential attachment procedure. The chromatic polynomial is computationally intensive to calculate, particularly for networks of large size, thus we restricted our analysis to networks of size 20. The rate of attachment affects the number of edges and therefore the density, so that $v = 3$ networks have higher density. They also have a higher chromatic number. In order for a solution to exist, nodes in networks set at $v = 3$ must have at least four specializations to choose from (Kearns, Suri, & Montfort 2006). The clustering coefficient is slightly higher for the networks with higher preferential attachment rates, and the average shortest path length is slightly lower. The average chromatic polynomial, i.e. the solution set, is much higher at higher rates of preferential attachment—in the order of 100 times higher. The standard deviation for the chromatic polynomial is also extremely high, indeed larger than the average in both network types.

Figure 6. Fraction of specialization by chromatic polynomial without property

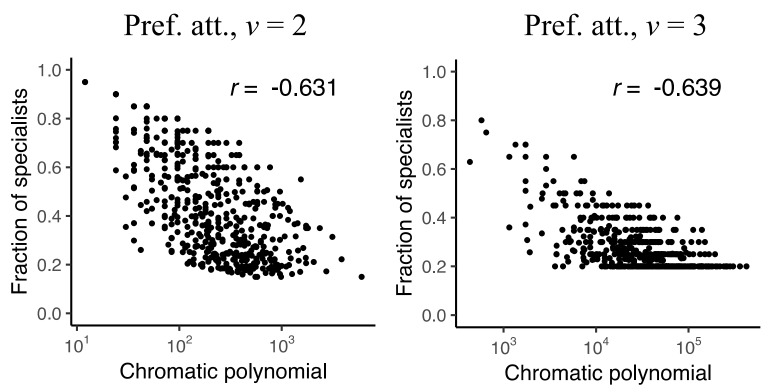


Figure 6 presents the first set of results for the simulation at the two different levels of attachment for networks operating in the condition of no property, meaning they have no storage capacity for goods (i.e., the threshold = 0 in Figure 3). The x-axis represents the chromatic polynomial with logarithmic conversion. The further left observations fall, the larger the number of solutions they have that exist to resolve the coordination problem. The y-axis is the fraction of the total population of nodes that adopt specialization by the end of the simulation. The *r* values indicate Pearson's product moment correlation coefficient between the logarithmic chromatic polynomial and the fraction of specialists.

Comparing the two graphs provides some initial information. Please note, the x-axis is shifted right for the *v*=3 network observations. Observations along this dimension begin about where they end for the *v*=2 networks. As is evident in the descriptive statistics, the average number of solutions to the complementarity problem is higher in these more dense and complex networks. These networks are less successful in transitioning to specialization. The fraction of population specialization across all the observations clusters at the lower levels.

The same general pattern is repeated within the attachment types. For networks with *v*=2, having more solutions to the complementarity problem produces lower rates of specialization. The largest proportion of the population adopts specialization when the number of solutions is lower. Similarly, for networks with *v*=3, larger proportions of the population transition to specialization when the chromatic polynomial is lower. The higher the chromatic polynomial goes, i.e. the larger the number of possible solutions, the less likely it is for agents to find those solutions.

Thus, in the condition of no property, the chromatic polynomial has a clear relationship to the emergence and spread of interdependent specialization; however, that association runs in an unexpected direction. While we might expect that having more solutions to a puzzle would make solving it easier, we instead observe that as the chromatic polynomial increases, the proportion of nodes that specialize decreases. This association cannot be explained by increased density as density is held constant across the nodes with different polynomials. It, however, can be explained by a breakdown in the structural constraint experienced by actors that, as shown in previous examples, can serve as a social guide. Without this guide, too many possible solutions lead to less-constrained local conditions where agents are not incentivized to specialize. A lack of constraint leads actors to get lost in a forest of possible pathways.

These findings, however, are strongly affected by the existence or non-existence of property. In Figure 7, the results are presented for preferential-attachment networks that exist under the condition of partial specialization, where property can be stored for later rounds of exchange (i.e., the threshold exceeds the agent's counter in Figure 3).

Figure 7. Speed of specialization by chromatic polynomial, partial specialization allowed

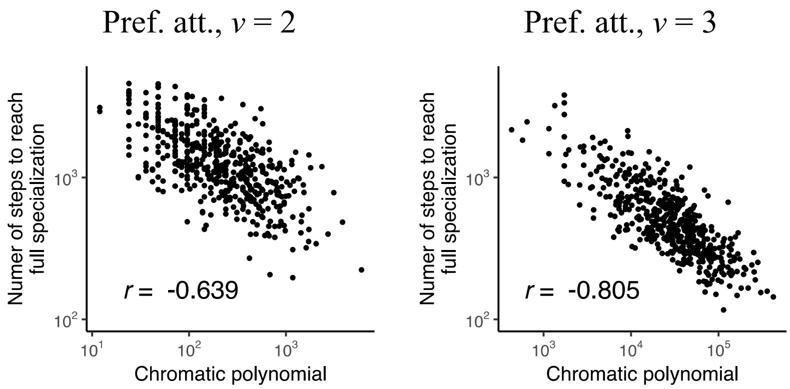


Figure 7 again presents two panels with networks of moderate and higher levels of preferential attachment. In the partial specialization condition with infinitely great threshold, where actors can safely store any amount of property, all networks ultimately achieve total specialization (i.e., the fraction of specialists = 1.0 at the end of all simulation sessions). From this outcome, it can be inferred that storage capacity and property rights appear to encourage the emergence of the division of labor in the more complex preferential attachment networks. Network structure, however, still matters in terms of the time it takes to achieve full specialization of the population.

The y-axis in this case is the number of steps it takes for the entire population to reach an optimal specialization (where there are no overlaps). The x-axis remains the chromatic polynomial with logarithmic conversion, and the axis is staggered across the panels, as with Figure 6, so that the right panel begins at a higher chromatic polynomial. The *r* values indicate Pearson's correlation coefficient between the logarithmic chromatic polynomial and the steps to an optimal specialization. The main difference in the panels, however, is that the implication of the y-axis runs in different direction than in Figure 6. The higher the y-value, the longer the time until optimal specialization. It follows then, that if we compare panels, under the partial specialization condition, specialization tends to occurs more quickly in the networks with higher levels of preferential attachment. Similarly, the chromatic polynomial now has a reversed effect from the world without property. Higher numbers of possible solutions lead more quickly to optimal specialization in both the moderate and high preferential attachment networks.

Protections for private property encourage the division of labor in these models. Property also mediates the impact of network structure. Where structural constraint in the complete specialization world acted as a social guide, the same constraint acts as a social trap in partial specialization. As we might expect, having more solutions makes it easier to solve the problem – but only when storage of property is possible.

Figure 8. Different specialization dynamics with three levels of chromatic polynomial.

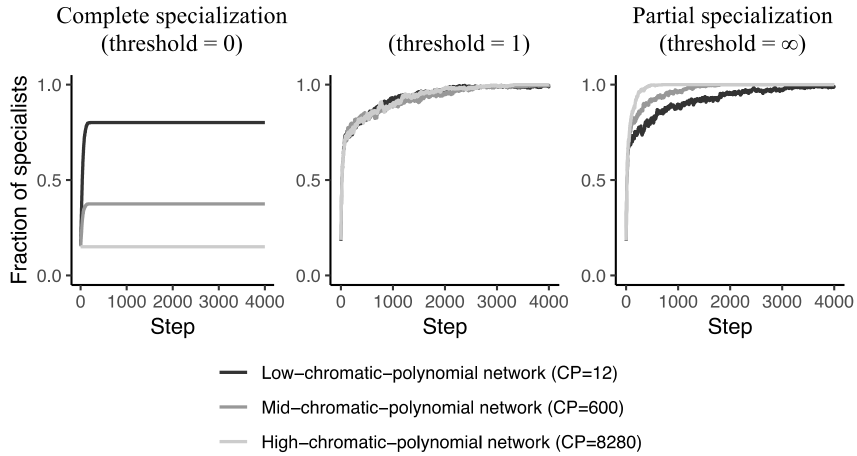


Figure 8 summarizes the impact of property rights by presenting the outcome for preferential attachment networks of *v*=2 with chromatic polynomials of 12, 600, and 8280 respectively (see Figure 1 for the topology of chromatic polynomial=12 and chromatic polynomial=8720 networks). The graphs show the average values of specialization rates for 100 repetitions over the session. As before, the outcome for complete specialization, no property, is measured in terms of proportion of the population that adopts specialization and for partial specialization, with property, the y-axis is time to full adoption. The threshold sets the storage capacity of actors, how much of a specialized good they can set aside for future rounds of exchange (Figure 3). The black lines refer to the network with a chromatic polynomial of 12, the grey lines to a network with a chromatic polynomial of 600, and the light grey lines refer to a network with a chromatic polynomial of 8,280. As is clear from the figure, the order of the lines is reversed from complete specialization when agents are allowed to store an unlimited amount of goods. The relationship of structural constraint is transformed from one in which constraint acts as a guide to on in which constraint acts as a sticky but not inescapable trap.

Conclusion:

Consistent with previous work, our findings do indeed suggest that network structure is an important factor in predicting whether groups will be able to successfully specialize in the interdependent manner necessary for achieving a productive division of labor. We think that two aspects of the research outcomes deserve particular attention.

The first aspect we would like to emphasize is the role of property protections. The existence of property rights is pivotal in both network models. In both types of networks, it completely transforms the impact of structural constraint. In networks high in structural constraint, property rights decrease the likelihood of specialization. But when small amounts of randomness are introduced to rigidly structured networks, property begins to increase the likelihood of specialization and the emergence of the division of labor. For more complex preferential attachment networks, complete specialization and the division of labor across the entire population become inevitable when the concept of property is introduced to the model. The benefits of property have often been conceived of as a way to encourage participation in economic growth through the incentive of wealth accumulation. These results suggest that another very significant benefit of property in societies without rigidly structured relational patterns may be the way in which it greatly assists in efforts to coordinate interdependence between economic agents. This effect could be party of the reason why property rights have been associated with economic growth (North 1990 1994, Rodrik et al. 2004, Acemoglu, Johnson, and Robinson 2001, 2002, Knack and Keefer 1995, Hall and Jones 1999).

The second aspect is the significance of the chromatic polynomial. The chromatic polynomial is difficult to measure and hard to observe, but important nonetheless because it is a shadow of the solution space of coordination concealed beneath network topology (Shirado and Christakis 2017). In statistical analysis of the association between the chromatic polynomial (available in the appendix), and fraction of the population that specializes in preferential advantage networks with complete specialization, it is significant even controlling for the clustering coefficient and shortest path length ($P < 0.001$ for each preferential-attachment model with linear regression; $N = 200$) and has a much larger effect size than the other two. The same is true for the number of steps it takes for the whole population to adopt specialization in the partial-specialization condition ($P < 0.001$ for each preferential-attachment model with linear regression; $N = 200$). The independent effect of the chromatic polynomial on the emergence of the division of labor suggests to us that this feature of networks may play an important an as yet unexamined role in other social processes – particularly coordinative and cooperative activities.

It is also interesting to note that we find that constraint is not always a negative. Structural constraint in these models serves as a guide as well as a potential trap. There are conditions under which constraint can help coordinate activity. And, simple networks with recognizably patterned features can encourage the development of specialization. These differences also highlight the importance of the temporal ordering of exchange relations. Sequentially and simultaneity, proxied in storage capacity or property regulations, operate very differently under similar structural conditions (Erikson 2018). In order to gauge the impact of network topology, it was necessary to hold structure constant (Shirado et. al. 2019, Shirado, Iosifidis & Christakis 2019). It would be worth considering how solutions vary in a mobile network in which nodes can dissolve and construct ties – although this would be asking a slightly different question than what we have posed here.

In interpreting the results, it is of clear importance to ask whether this modified graph-coloring game really captures enough of the complexities of real-world exchange to serve as a useful model for the evolution of a division of labor. The graph-coloring game is a stylized, conceptual model that ignores many factors, such as the uneven distribution of resources and skill. In particular it is unlikely to capture dynamics around the proliferation rather than emergence of specialization, which may depend upon the existence of already specialized brokers, substitutability, and mobility. All of these phenomena, however, could be explored using different versions of the adapted model in order to explore slightly different questions.

We would argue that our treatment of the model provides conceptual clarification and useful dialogue with previous theoretical treatments of the problem. Additionally, while we would agree that few contemporary societies completely lack specialization, there are areas of production in which the interdependency problems of specialization are more pronounced than others. Commodity chain production is one example. If one factory producing one specialized component fails to produce, the final product cannot be made. Each firm specializes in a different part and relies upon the other firm to make the other specialized parts. Perhaps more importantly, the firms are likely to be bound in a network of pre-existing relationships that condition their knowledge of and trust in other firms to fulfill their contracts. One might also, consider international vaccine production. Specialization in this case is both decentralized and highly interdependent – and constrained by patterns of alliance and conflict. More generally, monopoly and monopsony business models depend upon differentiation from competitors. Resolving the specialization problem thus is related to the problem of both horizontal integrations, of which monopolization is an extreme case, and vertical integration, or the internalization of commodity chain production. In general, there is agreement in the sociological literature on the division of labor that the phenomenon encompasses a broad set of interdependence problems not limited to sustenance (Kempner 1972: 743, Udy 1959, Gibbs & Poston 1975). The continued existence of these problems of interdependency suggest that there is good reason for continuing to think through the emergence of the division of labor through a theoretical lens.

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