Misperceiving Interactions among Complements and Substitutes

(technical Appendix)

Nicolaj Siggelkow
Wharton School
University of Pennsylvania

This note contains additional details for the proofs of Propositions 7 and 10, and material noted in footnote 8 that were omitted from the main part of the paper.

General set-up: In the additive case (AC), a manager perceives the interaction to be $(\alpha + \delta)$ rather than α. In the multiplicative case (MC), the manager perceives $(1 + \delta)\alpha$. For second-order conditions to be fulfilled, it is assumed that $|\alpha| < 2$, $|\alpha + \delta| < 2$ (for AC), and $|(1+\delta)\alpha| < 2$ (for MC). We also focus on relationship- conserving misperceptions, i.e., $\alpha + \delta > 0$ if $\alpha > 0$, and $\alpha + \delta < 0$ if $\alpha < 0$ (for AC) and $\delta \geq -1$ (for MC).

For compactness, it is useful to compute the expressions for the performance declines for the general two-manager case and treat the one-manager case as a special case with $\delta = \delta_1 = \delta_2$.

For AC: $$R(\alpha, \delta_1, \delta_2) = -\frac{(2+\alpha)\delta_2^2 + (2+\alpha)\delta_1\delta_2(\alpha+\delta_2) + \delta_1^2(2+\alpha+2\delta_2+\alpha\delta_2+\delta_2^2)}{(2-\alpha)(4-(\alpha+\delta_1)(\alpha+\delta_2))^2} \qquad \text{(A1)}$$

For MC: $$R(\alpha, \delta_1, \delta_2) = -\frac{\alpha^2((2+\alpha)\delta_2^2 + \alpha(2+\alpha)\delta_1\delta_2(1+\delta_2) + \delta_1^2(2+\alpha(1+\delta_2(2+\alpha+\alpha\delta_2))))}{(2-\alpha)(4-\alpha^2(1+\delta_1)(1+\delta_2))^2} \qquad \text{(A2)}$$

Proposition 7: *If the interaction parameter α is assumed to be randomly distributed, a firm will choose a lower variance of the signal (lower level of d) if $0 \leq \alpha \leq 1$ than if $-1 \leq \alpha \leq 0$.*

Denote with $g(\alpha|z)$ the conditional distribution of α given signal z. Given a quadratic production function and a signal z, the manager maximizes over her choices A and B

$$V(z) = \int (A + B + \alpha AB - A^2 - B^2)\, g(\alpha|z)\, d\alpha \qquad \text{(A3)}$$

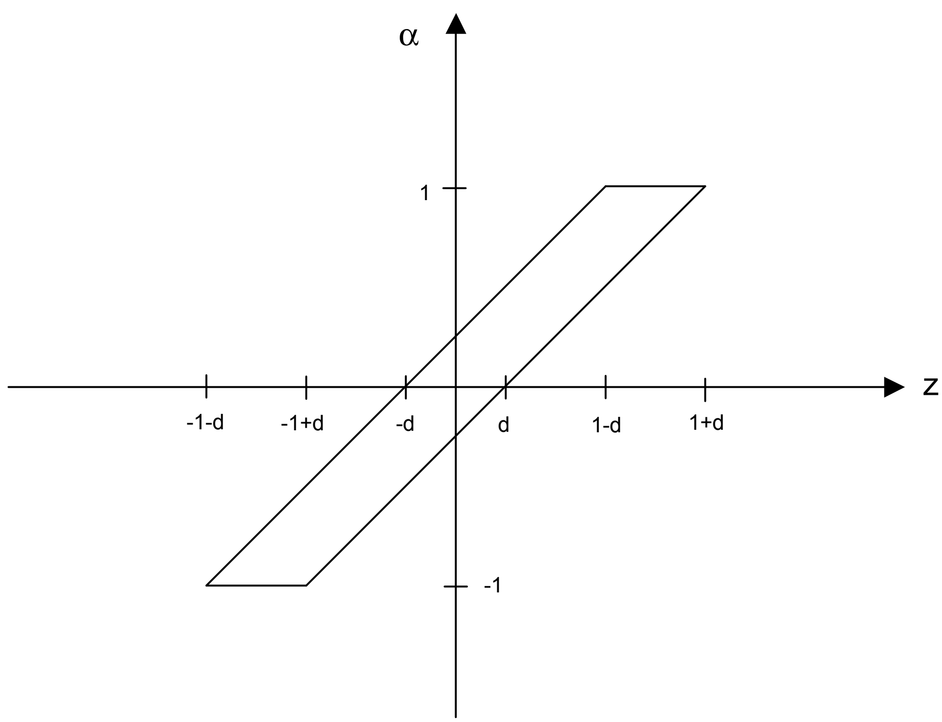
Given optimal choices A^* and B^*, i.e. solutions to (A3), the expected gross benefit of choosing a precision d of the signal is:

$$E(V^*(d)) = \iint (A^* + B^* + \alpha A^* B^* - A^{*2} - B^{*2}) g(\alpha \mid z) h(z)\, d\alpha dz \qquad \text{(A4)}$$

where $h(z)$ is the probability density function of z.

Figures A-1 and A-2 are helpful in determining the correct integration bounds. The figures also show that the geometry of the integration spaces changes between the cases $d \in [0, ½]$ and $d \in [½, 1]$. Hence, it is helpful to distinguish between these two cases.

Figure A- 1: Plot of possible (α, z) combinations if $0 \le d \le 1/2$



For illustration, let us consider the case $\alpha > 0$ and $d \in [0, ½]$. As Figure A-1 illustrates, in this case z lies between $-d$ and $1 + d$.

If

a) $-d \le z \le d$, then $\alpha \sim$ u$[0, z+d]$ and $g(\alpha/z) = 1/(z+d)$; and $h(z) = ½ + z/2d$

b) $d \le z \le 1-d$, then $\alpha \sim$ u$[z-d, z+d]$ and $g(\alpha/z) = 1/(2d)$; and $h(z) = 1$

c) $1-d \le z \le 1+d$, then $\alpha \sim$ u$[z-d, 1]$ and $g(\alpha/z) = 1/(1+d-z)$; and $h(z) = ½ + (1-z)/2d$

For case a) equation (A3) can be written as:

$$\int_0^{z+d} (A + B + \alpha AB - A^2 - B^2)(1/(z+d)d\alpha = A + B + \frac{1}{2}(z+d)AB - A^2 - B^2 \qquad \text{(A5)}$$

Maximizing (A5) with respect to A and B, yields optimal choices

$A^* = B^* = 2/(4 - d - z)$

Substituting these values into the production function, yields

$$V^* = \frac{4(2 + \alpha - d - z)}{(4 - d - z)^2} \qquad \text{(A6)}$$

Substituting (A6) into (A4) yields:

$$E_1 = \int_{-d}^{d} \int_0^{z+d} \frac{4(2 + \alpha - d - z)}{(4 - d - z)^2}\left(\frac{1}{z+d}\right)\left(\frac{z+d}{2d}\right) d\alpha dz = \frac{ln(16) - 2d - 4\,ln(2-d)}{d}$$

Going through the same steps for sub–cases b) and c) yields:

$$E_2 = \ln(2-d) - \ln(1+d)$$

$$E_3 = \frac{2d - 2\ln(1+d)}{d}$$

Thus, the expected value achieved if the precision is d, equals to

$E(V^*(d)) = E_1 + E_2 + E_3.$

Lastly, we differentiate $E(V^*(d))$ with respect to d:

$$\frac{\partial E(V^*(d))}{\partial d} = -\frac{1}{d^2}\left(\frac{3d^2}{d^2 - d - 2} + \ln(16) - 4\ln(2-d) - 2\ln(1+d)\right) \qquad \text{(A7)}$$

To complete the analysis for complements, we repeat the above steps for the case $\alpha > 0$ and $d \in [½, 1]$. We then repeat the steps for the case of $\alpha < 0$. Lastly, we plot the partial derivatives in Figure A-3. As the figure shows, the marginal benefit of decreasing *d* is always larger in the case of complements than in the case of substitutes. As a result, for any given convex cost function *c*, the optimal level of imprecision concerning the interaction parameter is higher in the case of complements than in the case of substitutes (one representative example of *c'* and the resulting optimal choices *d** are illustrated in the figure). In other words, a firm will (optimally) invest more to reduce uncertainty around complementary interactions than around substitute interactions.

Figure A-2: Plot of possible (α, *z*) combinations if $1/2 \leq d \leq 1$

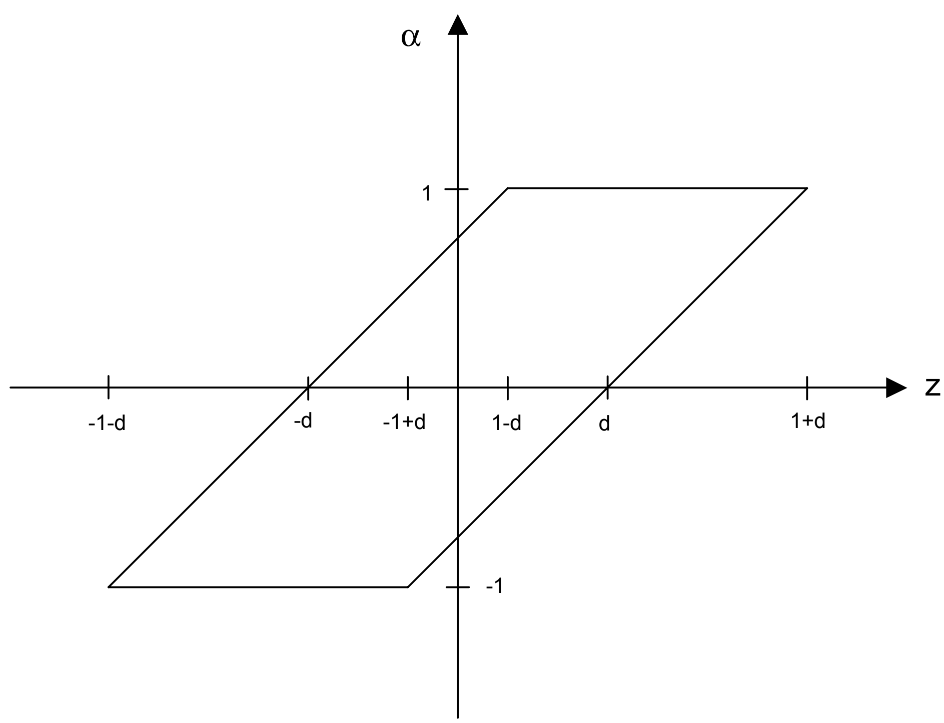
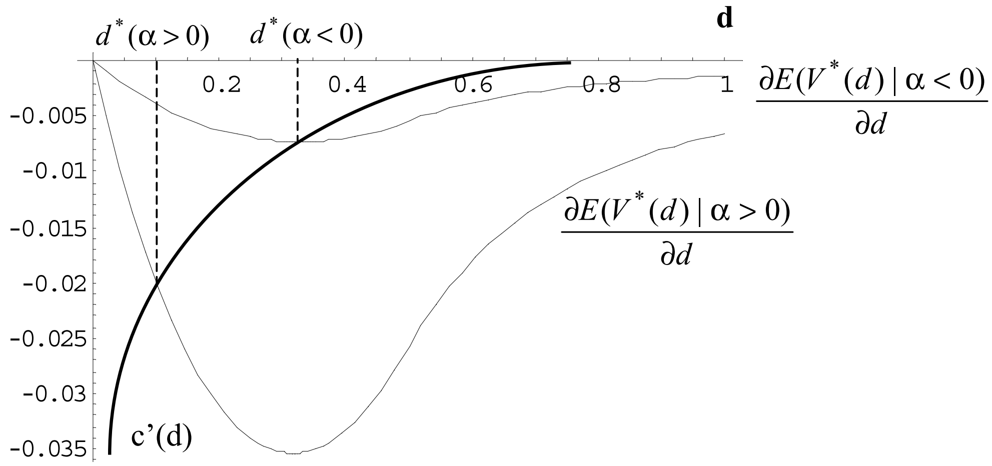


Figure A-3: Plot of marginal benefits of changing *d*, for complements and substitutes, one illustrative cost curve and the resulting optimal levels of *d*



Proposition 10: *Propositions 4–6 hold with M1 having no misperception*.

A similar approach as in proving Propositions 4–6 is followed. We need to show:

for AC: $R(-\alpha, 0, \delta_2) - R(\alpha, 0, -\delta_2) > 0$ for $\alpha > 0$ (A8)

for MC: $R(-\alpha, 0, \delta_2) - R(\alpha, 0, \delta_2) > 0$ for $\alpha > 0$ (A9)

Evaluating (A8) yields: $$\frac{(2+\alpha)\delta_2^2}{(2-\alpha)(4-\alpha^2+\alpha\delta_2)^2} - \frac{(2-\alpha)\delta_2^2}{(2+\alpha)(4-\alpha^2+\alpha\delta_2)^2} \quad \text{(A10)}$$

Evaluating (A9) yields: $$\frac{\alpha^2(2+\alpha)\delta_2^2}{(2-\alpha)(4-\alpha^2(1+\delta_2))^2} - \frac{\alpha^2(2-\alpha)\delta_2^2}{(2+\alpha)(4-\alpha^2(1+\delta_2))^2} \quad \text{(A11)}$$

In both (A10) and (A11) the first term has a larger positive numerator and a smaller positive denominator for all δ. Hence, both (A10) and (A11) are positive for all δ. □

Footnote 8: The value of information

We can assess the value of informing M1 about the true interaction by comparing the outcome of this regime with the outcome resulting from the case in which both M1 and M2 have a misperception. A natural benchmark for comparison is the case of symmetric misperception (i.e., $\delta = \delta_1 = \delta_2$). Formally,

$V(\alpha, 0, \delta)$, the resulting value when M1 has no misperception, is compared to $V(\alpha, \delta, \delta)$, the resulting value when both managers have equal misperception. The difference

$$T(\alpha, \delta) = V(\alpha, 0, \delta) - V(\alpha, \delta, \delta) \tag{A12}$$

captures the value of reducing M1's misperception from δ to zero. (Note, if M1 can transmit the information about the true interaction to M2, the analysis of the one-manager case applies, since in this case $T(\alpha, \delta) = V(\alpha, 0, 0) - V(\alpha, \delta, \delta) = -R(\alpha, \delta)$.)

While the value of information is generally positive, it is interesting to note that conditions exist under which total value is higher when M1 has the same faulty perception of the interaction than when she has the correct perception.

Proposition 11: *Informing one manager about the correct interaction strength while the other manager still has misperception δ can lead to a larger performance decline than having both managers with the same misperception δ.*

Proof: We focus on AC. The value of informing one manager is given by:

$$T(\alpha, \delta) = R(\alpha, 0, \delta) - R(\alpha, \delta, \delta) = \frac{-\delta^2(-4 - 4\alpha + \alpha^2 + \alpha^3 + 2(\alpha^2 + \alpha - 2)\delta + (1+\alpha)\delta^2)}{(2 - \alpha - \delta)^2(4 - \alpha(\alpha + \delta))^2} \tag{A13}$$

Setting (A25) equal to zero and solving for δ yields: $\delta' = \dfrac{2 - \alpha - \alpha^2 - 2\sqrt{2+\alpha}}{1+\alpha}$ (A14)

For any $\delta < \delta'$ the value of information is negative, i.e., $T(\alpha, \delta) < 0$.

To gain intuition for when Proposition 11 holds, it is helpful to compute the optimal misperception for M1. Formally, we maximize $V(\alpha, \delta_1, \delta_2)$ with respect to δ_1 given α and δ_2. It turns out that the optimal misperception for M1, $\delta_1^*(\delta_2)$, is always negative, except for values of δ_2 lying between 0 and $-\alpha$. (The choices that result from this belief are equivalent to the choices that would result if M1 could move first and commit to the choice of A, i.e., if M1 were a Stackelberg leader.) Figure A-5 provides an example for $\alpha = -½$.

The curved line in Figure A-5 depicts the optimal misperception of M1 as a function of M2's misperception. The diagonal line describes the case in which both managers have the same misperception, $\delta_1 = \delta_2$. The horizontal axis corresponds to M1 knowing the true interaction ($\delta_1 = 0$). As Figure A-5 shows, if both managers have very negative δ's, the optimal misperception of M1 lies closer to the actual misperception than to the truth. It is in this case, that equal misperception of M1 and M2 yields a better outcome than M1 perceiving the true interaction. Furthermore, as Figure A-5 illustrates, for M1 to perceive the true interaction is optimal in only two situations: First, when M2 has no misperception. Second, when M2 ignores the interaction, i.e., when $\delta_2 = -\alpha$. In the latter case, since M2 believes that the marginal benefit of *B* is not affected by M1's decision, M1 cannot influence M2's choice. Hence, the best M1 can do is act according to the true interaction.

Figure A-5: Optimal misperception vs. no and equal misperceptions